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Transport coefficients of a quantum system interacting with a squeezed heat bath

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The analytical expressions for the time-dependent friction and diffusion coefficients are presented for the case of coupling in coordinates between the collective subsystem and a squeezed heat bath. The effects of initial phase-sensitive and -insensitive correlations of the heat bath on the diffusion coefficients, fluctuations, and decoherence are studied. The interplay between friction and decoherence is discussed.

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I. INTRODUCTION

The observation of quantum-mechanical features in the macroscopic systems is of fundamental interest. There have been considerable attempts in finding macroscopic quantum coherence in atomic and condensed matter physics [1–8,10–15]. The methods have been suggested to create a quantum superposition state in optical systems. Nonclassical optical effects such as quadrature squeezing, higher-order squeezing, oscillations in photon number distribution, and so on have their origin in quantum interference phenomenon. The main problem with the observation of a macroscopic superposition of quantum states is that the macroscopic objects are not isolated but coupled with an environment which can be modeled as a heat bath. It has been shown in Ref. [3] that the quantum coherence is destroyed very fast in the macrosystems under the influence of a phase-insensitive thermal heat bath. The authors of Ref. [3] have found that the macroscopic superposition of states of light is preserved in the presence of dissipation when the quantum fluctuations of the heat bath are squeezed. They have shown that the squeezed bath is more efficient than the thermal bath for optical quadrature-phase quantum measurements, and may also be used to prepare the states with low quantum noise in one quadrature phase, at least in the high-frequency regime. It has been found in Ref. [4] that the decay rate of quantum coherence in the phase-sensitive heat bath can be significantly modified compared to the decay rate in the ordinary phase-insensitive thermal bath. Depending on the phases of the quantum system (field mode) and the squeezed reservoir, the decay rate of the quantum coherence can be either enhanced or significantly suppressed. Suppression of the decay rate of the quantum coherence leads to the preservation of nonclassical effects such as the oscillations in the photon number distribution. The description of the mentioned processes is based on the quantum master equation or Fokker-Planck equation for the Wigner function in phase space [16,17]. These equations are derived under the rotating wave, Born, and Markov approximations.

The functional integral formalism has been used in Refs. [7,15] to provide an exact treatment of the effect of squeezed bath on the decoherence and dissipation properties of an open quantum system. Using the Ohmic bath and the high-temperature limit, the coefficients of the master equation describing dissipation and decoherence have been obtained and analyzed in Ref. [15].

In our paper the effect of the squeezed bath on the collective system is studied with the Langevin formalism which is widely applied to find the effects of fluctuations and dissipations in macroscopical systems. The Langevin method in the kinetic theory significantly simplifies the calculation of nonequilibrium quantum and thermal fluctuations and provides a clear picture of the dynamics of the process [13,14,18]. As in Refs. [19,20], we derive exactly the equations of motion for the time-dependent open system and bath. Then we obtain the analytical expressions for the time-dependent transport coefficients of the system (harmonic oscillator) linearly coupled to a squeezed bath of oscillators. These transport coefficients contain the non-Markovian effects which can be very important in new devices needed for the emerging fields of nanotechnology and quantum computing. The decoherence is studied by analyzing the magnitude of diffusion coefficients in the quantum master or Fokker-Planck equation. With our rigorous formalism one can describe the nonequilibrium quantum statistical processes beyond the weak-coupling and high temperature limits.

In Sec. II we define the model and present the quantum-mechanical derivation of generalized Langevin equations for the fully coupled oscillator (FC oscillator) interacting with the heat bath. We consider the general case when the bath is initially squeezed. The results of illustrative numerical calculations of diffusion and friction coefficients, uncertainty relation, decoherence, and variances are presented in Sec. III for different strength of the damping and temperatures. We apply our formalism to the nuclear systems and consider the quadrupole deformation parameter of the nucleus surface as the collective coordinate. In this case the heat bath consists of internal degrees of freedom.

II. GENERALIZED FC OSCILLATOR

In order to derive the quantum non-Markovian Langevin equations and the time-dependent transport coefficients for a collective subsystem, the suitable microscopic Hamiltonian of the whole system (the heat bath plus collective system) has to be first formulated. In Refs. [21–23] the following Hamiltonian was suggested:

$$H = H_c + H_b + H_{cb},$$

$$H_c = p\frac{1}{2\mu}p + U(q),$$

$$H_b = \sum_\nu \hbar\omega_\nu b_\nu^+ b_\nu,$$

$$H_{cb} = \sum_\nu V_\nu(q)(b_\nu^+ + b_\nu). \tag{1}$$

which explicitly depends on the collective coordinate q, canonically conjugated momentum p, and intrinsic heat bath degrees of freedom expressed through the bath phonon operators of creation b_ν^+ and annihilation b_ν. The terms H_c, H_b, and H_{cb} are the Hamiltonians of the collective subsystem depending on mass parameter μ and potential $U(q)$, of the bath subsystem and of the collective-bath interaction, respectively. The heat bath is modeled by an assembly of harmonic oscillators with frequencies ω_ν. The coupling to the heat bath is linear in the bath operators and corresponds to the energy being transfered to and from the bath by the absorption or emission of bath quanta. The coupling term has important influence on the dynamics of the collective subsystem, by altering the effective collective potential and by allowing the energy to be exchanged with the thermal reservoir, thereby allowing the subsystem to attain the thermal equilibrium with the heat bath.

For the FC oscillator with $U(q)=\mu\omega^2 q^2/2$, the coupling Hamiltonian is [2]

$$H_{cb} = \lambda^{1/2}\sum_\nu \Gamma_\nu (a^+ + a)(b_\nu^+ + b_\nu) = \sqrt{\frac{2\lambda\mu\omega}{\hbar}}\, q \sum_\nu \Gamma_\nu (b_\nu^+ + b_\nu). \tag{2}$$

Here, a^+ and a are the annihilation and creation operators of the quantum oscillator, respectively, Γ_ν real coupling constants and λ the parameter which measures the average strength of interaction. As in Ref. [19], we write a set of the Langevin equations for the damped quantum FC oscillator,

$$\dot q(t) = \frac{p(t)}{\mu},$$

$$\dot p(t) = -\tilde\delta q(t) - \kappa^2 \int_0^t d_\tau K(t-\tau)\dot q(\tau) + \kappa F(t), \tag{3}$$

where the operator of random forces in momentum,

$$F(t) = -\frac{\lambda^{1/2}}{\hbar}\sum_\nu \Gamma_\nu [f_\nu^+(t) + f_\nu(t)],$$

$$f_\nu(t) = \left(b_\nu(0) + \frac{\kappa\lambda^{1/2}\Gamma_\nu}{\hbar^2\omega_\nu} q(0)\right) e^{-i\omega_\nu t},$$

$$\kappa = (2\mu\omega\hbar)^{1/2},$$

the renormalized stiffness,

$$\tilde\delta = \mu\omega^2 - \frac{2\lambda\kappa^2}{\hbar^2}\sum_\nu \frac{\Gamma_\nu^2}{\hbar\omega_\nu},$$

and the dissipative kernel,

$$K(t-\tau) = \frac{2\lambda}{\hbar^2}\sum_\nu \frac{\Gamma_\nu^2}{\hbar\omega_\nu}\cos(\omega_\nu[t-\tau]).$$

In comparison with Refs. [2,11] we do not introduce the counterterm in the Hamiltonian in order to compensate the renormalization of the potential. Since the asymptotic stiffness corresponds to the observable, the parameters in our approach are adjusted so to have certain fixed value of the asymptotic stiffness. So, the coupling to the heat bath introduces only dissipation and fluctuation.

Equations (3) have the following solutions:

$$q(t) = A_t q(0) + B_t p(0) + \kappa\int_0^t d\tau B F(t-\tau),$$

$$p(t) = -\mu\tilde\delta B_t q(0) + \mu\dot B_t p(0) + \mu\kappa\int_0^t d\tau \dot B F(t-\tau), \tag{4}$$

where

$$B_t = \frac{1}{\mu}\mathcal{L}^{-1}\left[\frac{1}{s^2 + 2\hbar\omega s K(s) + \tilde\delta/\mu}\right],$$

$$A_t = \mu\dot B_t + \kappa^2\int_0^t d\tau B_\tau K(t-\tau) = \mathcal{L}^{-1}\left[\frac{s + 2\hbar\omega K(s)}{s^2 + 2\hbar\omega s K(s) + \tilde\delta/\mu}\right]. \tag{5}$$

Here, $K(s)=\mathcal{L}[K(t)]$ is the Laplace transformation of $K(t)$ and $\mathcal{L}^{-1}$ denotes the inverse Laplace transformation. Since the kern K does not contain the phonon occupation numbers, it is independent of the temperature T of the heat bath. The temperature and fluctuations enter in the analysis through the specification of the distribution of initial conditions.

In order to calculate the correlation functions of the fluctuations and to check the dependence of transport coefficients on the statistics of thermal heat bath, we consider two statistics:

$$\langle\langle f_\nu^+(t) f_{\nu'}^+(t')\rangle\rangle = \langle\langle f_\nu(t) f_{\nu'}(t')\rangle\rangle = 0,$$

$$\langle\langle f_\nu^+(t) f_{\nu'}(t')\rangle\rangle = \delta_{\nu,\nu'} n_\nu e^{i\omega_\nu(t-t')},$$

$$\langle\langle f_\nu(t) f_{\nu'}^+(t')\rangle\rangle = \delta_{\nu,\nu'} (n_\nu + 1) e^{-i\omega_\nu(t-t')} \tag{6}$$

for the uncorrelated thermal bath and

$$\langle\langle f_\nu^+(t) f_{\nu'}^+(t')\rangle\rangle = \delta_{\nu,\nu'} m_\nu^* e^{i\omega_\nu(t'+t)},$$

$$\langle\langle f_\nu(t) f_{\nu'}(t')\rangle\rangle = \delta_{\nu,\nu'} m_\nu e^{-i\omega_\nu(t'+t)},$$

$$\langle\langle f_\nu^{+}(t) f_{\nu'}(t')\rangle\rangle = \delta_{\nu,\nu'} n_\nu e^{i\omega_\nu (t-t')},$$

$$\langle\langle f_\nu(t) f_{\nu'}^{+}(t')\rangle\rangle = \delta_{\nu,\nu'} (n_\nu + 1) e^{-i\omega_\nu (t-t')} \qquad (7)$$

for the correlated phase-sensitive thermal bath. Here, the symbol $\langle\langle \cdots \rangle\rangle$ denotes the average over the bath. The heat baths are characterized by the phonon number $n_\nu = [\exp(\hbar\omega_\nu/T)-1]^{-1}$ of a field mode with particular frequency and by the correlation $m_\nu = |m_\nu| e^{-i\phi_\nu}$ between the modes symmetrically displaced around this center frequency. The parameter ϕ_ν is the squeezing phase. The correlated maximally squeezed heat bath with $|m_\nu|^2 = n_\nu(n_\nu+1)$ [13,24] is used in the present paper. We recover the uncorrelated thermal bath in the limit $m_\nu = 0$. The occupation numbers n_ν for the phonons depend on T. The set up of the squeezed heat bath can be achieved by using the thermal bath in the external nonlinear and nonstationary electric field [10,13].

It is convenient to introduce the spectral density $D(\omega_0)$ of the heat bath excitations which allows us to replace the sum over different oscillators ν by an integral over the frequency: $\Sigma_\nu \cdots \rightarrow \int_0^\infty d\omega_0 D(\omega_0) \cdots$ and $\Gamma_\nu \rightarrow \Gamma_{\omega_0}$, $\omega_\nu \rightarrow \omega_0$, $n_\nu \rightarrow n_{\omega_0}$, $m_\nu \rightarrow m_{\omega_0}$, $\phi_\nu \rightarrow \phi_{\omega_0}$. Let us consider the following spectral functions [25]:

$$D(\omega_0) \frac{|\Gamma_{\omega_0}|^2}{\hbar^2 \omega_0} = \frac{1}{\pi} \frac{\gamma^2}{\gamma^2 + \omega_0^2},$$

where the memory time γ^{-1} of the dissipation is inverse to the phonon bandwidth of the heat bath excitations which are coupled with the collective oscillator. This is the Ohmic dissipation with a Lorenzian cutoff (Drude dissipation) [11,13,18,25]. The relaxation time of the heat bath should be much less than the period of the collective oscillator, i.e., $\gamma \gg \omega$.

If we replace the sums Σ_ν by the integrals over the bath frequencies, then

$$K(t) = \frac{\lambda\gamma}{\hbar} e^{-\gamma|t|},$$

$$K(s) = \frac{\lambda\gamma}{\hbar(s+\gamma)},$$

$$\tilde{\delta} = \mu(\omega^2 - 2\lambda\gamma\omega),$$

$$B_t = \sum_{i=1}^{3} B_t^i = \frac{1}{\mu} \sum_{i=1}^{3} \beta_i (s_i + \gamma) e^{s_i t},$$

$$A_t = \sum_{i=1}^{3} \beta_i [s_i(s_i+\gamma) + 2\omega\lambda\gamma] e^{s_i t},$$

where $\beta_1 = [(s_1 - s_2)(s_1 - s_3)]^{-1}$, $\beta_2 = [(s_2 - s_1)(s_2 - s_3)]^{-1}$, $\beta_3 = [(s_3 - s_1)(s_3 - s_2)]^{-1}$, and s_i $(i=1,2,3)$ are the roots of the cubic equation

$$\begin{aligned} d(s) &= s^2 + 2\hbar\omega s K(s) + \tilde{\delta}/\mu \\ &= [(s+\gamma)(s^2+\omega^2) - 2\omega\lambda\gamma^2]/(s+\gamma) = 0. \end{aligned} \qquad (8)$$

At $\gamma \rightarrow \infty$ we have the instantaneous dissipation with the kernel $K(t) = \frac{2\lambda}{\hbar}\delta(t)$.

In order to determine the friction and diffusion coefficients, we consider the equations for the first and second moments [19]. Making derivative in t of $p(t)$ and $q(t)$ in Eq. (4) and simple but tedious algebra, we obtain the following equations for the first moments in the coordinate $\langle q(t)\rangle$ and in the momentum $\langle p(t)\rangle$:

$$\frac{d}{dt}\langle q(t)\rangle = \frac{1}{\mu}\langle p(t)\rangle,$$

$$\frac{d}{dt}\langle p(t)\rangle = -\xi(t)\langle q(t)\rangle - \lambda_p(t)\langle p(t)\rangle \qquad (9)$$

and for the second moments (variances) in the coordinate $\sigma_{qq}(t) = \langle q^2(t)\rangle - \langle q(t)\rangle^2$, in the momentum $\sigma_{pp}(t) = \langle p^2(t)\rangle - \langle p(t)\rangle^2$, and in the mixed coordinate momentum $\sigma_{qp}(t) = \frac{1}{2}\langle p(t)q(t) + q(t)p(t)\rangle - \langle p(t)\rangle\langle q(t)\rangle$:

$$\dot{\sigma}_{qq}(t) = \frac{2}{\mu}\sigma_{qp}(t),$$

$$\dot{\sigma}_{pp}(t) = -2\lambda_p(t)\sigma_{pp}(t) - 2\xi(t)\sigma_{qp}(t) + 2D_{pp}(t),$$

$$\dot{\sigma}_{qp}(t) = -\lambda_p(t)\sigma_{qp}(t) - \xi(t)\sigma_{qq}(t) + \frac{1}{\mu}\sigma_{pp}(t) + 2D_{qp}(t). \qquad (10)$$

From the structure of equations (9) and (10) it is seen that the dynamics of system is determined by the nonstationary friction coefficients $\lambda_p(t)$ in the momentum

$$\lambda_p(t) = -\frac{A_t \ddot{B}_t + \tilde{\delta} B_t \dot{B}_t}{\tilde{\delta}(B_t)^2 + A_t \dot{B}_t}, \qquad (11)$$

the stiffness coefficient

$$\xi(t) = \tilde{\delta} \frac{(\dot{B}_t)^2 - B_t \ddot{B}_t}{\tilde{\delta}(B_t)^2 + A_t \dot{B}_t}, \qquad (12)$$

and the diffusion coefficients in momentum

$$D_{pp}(t) = \lambda_p(t) J_{p_t p_t} + \frac{1}{2}[\dot{J}_{p_t p_t} + \mu\xi(t)\dot{J}_{q_t q_t}] \qquad (13)$$

and the mixed coordinate-momentum diffusion coefficient

$$D_{qp}(t) = \frac{1}{2}\left[\xi(t) J_{q_t q_t} - \frac{1}{\mu} J_{p_t p_t} + \frac{\mu}{2}[\lambda_p(t)\dot{J}_{q_t q_t} + \ddot{J}_{q_t q_t}]\right], \qquad (14)$$

where

$$J_{q_t q_t} = \kappa^2 \int_0^t \int_0^t d d\tau' B_\tau B_{\tau'} \langle F(t-\tau) F(t-\tau')\rangle,$$

$$J_{p_tp_t} = (\mu\kappa)^2 \int_0^t \int_0^t d\tau d\tau' \dot{B}_\tau \dot{B}_{\tau'} \langle F(t-\tau)F(t-\tau')\rangle. \quad (15)$$

Here, the symbol $\langle\cdots\rangle$ means an verage over the whole system, and $\ddot{J}_{q_tq_t}=\frac{d^2J_{q_tq_t}}{dt^2}$, $\dot{J}_{q_tq_t}=\frac{dJ_{q_tq_t}}{dt}$ and $\dot{J}_{p_tp_t}=\frac{dJ_{p_tp_t}}{dt}$. In the Appendix we present the explicit expressions for $J_{q_tq_t}=J^{(1)}_{q_tq_t}+J^{(2)}_{q_tq_t}$ and $J_{p_tp_t}=J^{(1)}_{p_tp_t}+J^{(2)}_{p_tp_t}$ which include the thermal terms $J^{(1)}_{i_ti_t}$ and the terms $J^{(2)}_{i_ti_t}$ related to the squeeze of the bath states. The diffusion D_{qq} and friction λ_q coefficients in coordinate are equal to zero in Eqs. (9) and (10) because of the absence of the random forces in coordinate. Therefore we have obtained the Markovian-type (local in time) equations for the first and second moments, but with a general form of transport coefficients which explicitly depend on time. It can be shown that the appropriate equilibrium distribution is achieved in the course of the time evolution. At $t\to\infty$ the system reaches the equilibrium state $[\dot{\sigma}_{pp}(\infty)=\dot{\sigma}_{qq}(\infty)=\dot{\sigma}_{qp}(\infty)=0]$ and the asymptotic diffusion coefficients can be derived from Eq. (10):

$$D_{pp}(\infty) = \lambda_p(\infty)\sigma_{pp}(\infty),$$

$$D_{qp}(\infty) = \frac{1}{2}\left(\xi(\infty)\sigma_{qq}(\infty) - \frac{1}{\mu}\sigma_{pp}(\infty)\right). \quad (16)$$

Comparing Eqs. (13), (14), and (10), we obtain that $\sigma_{qq}(\infty)=J^{(1)}_{q_\infty q_\infty}$, $\sigma_{pp}(\infty)=J^{(1)}_{p_\infty p_\infty}$, and $\sigma_{qp}(\infty)=0$. As follows from the expressions in the Appendix, $J^{(2)}_{i_\infty i_\infty}=0$. Since $J^{(1)}_{i_ti_t}$ contains the time-independent term, $J^{(1)}_{i_\infty i_\infty}\neq 0$. The asymptotic diffusion and friction coefficients in momentum are connected by the well-known fluctuation-dissipation relation.

III. RESULTS OF CALCULATIONS

As was shown, the diffusion and friction coefficients depend on the parameters ω, λ, and γ. The values of ω and λ are fixed by given certain asymptotic values of $\xi(\infty)$ and $\lambda_p(\infty)$,

$$\xi = \xi(\infty) = \mu\tilde{\omega}^2, \quad \lambda_p(\infty) = \lambda_p.$$

In the present paper the dynamical collective variable is the quadrupole deformation parameter which describes the surface vibrations in a heavy atomic nucleus [31]. The motion in quadrupole deformation parameter produces small oscillations around the equilibrium nuclear shape. The excited vibrational nuclear states are populated in the deep-inelastic collisions between heavy ions. One can measure the widths (a few MeV), which are related to the friction coefficient λ_p, of these states and the transitions from these states to other nuclear states. We choose characteristic frequency $\hbar\tilde{\omega}=1$ MeV and mass parameter $\mu=448m_0$ (m_0 is the nucleon mass) for this type of collective quantum motion. The motion of nuclear system in mass asymmetry coordinate has the similar characteristics [32]. We fix the frequency of collective oscillator and express all energetic characteristics in the units of this frequency. The following values of friction coefficients $\lambda_p=0.5\tilde{\omega}$ (underdamped limit) and $\lambda_p=2\tilde{\omega}$ are considered. The parameters $(\omega/\tilde{\omega},\lambda)=(4.6,0.20)$, (3.5, 0.13), and (2.6, 0.09) correspond to $\lambda_p/\tilde{\omega}=2$, 1, and 0.5, respectively. Since the effect of squeezing increases with temperature, the calculations are performed for $T=\hbar\tilde{\omega}$ and $T=2\hbar\tilde{\omega}$. The value of γ should be taken to hold the condition $\gamma\gg\omega$. We set $\gamma/\tilde{\omega}=12$. The role of value of $\phi_{\omega_0}=0$ is sampled.

A. The case of $\phi_{\omega_0}=0$

Let us consider the effect of $m_{\omega_0}\neq 0$ ($\phi_{\omega_0}=0$) on the value of transport coefficients. The time evolutions of diffusion coefficients $D_{pp}(t)$, $D_{qp}(t)$, and the friction coefficient $\lambda_p(t)$ are shown in Fig. 1. These coefficients are equal to zero at initial time and after some transient time reach the asymptotic values. The transient time increases with λ_p. During short initial time the value of D_{pq} is positive and becomes negative later on. The asymptotic value of D_{qp} goes to zero with increasing temperature and decreasing λ_p. The high temperatures and small friction coefficients correspond to the classical limit. The friction coefficient is temperature independent. Comparing the results of calculations with $m_{\omega_0}\neq 0$ and $m_{\omega_0}=0$, we demonstrate that the effect of squeezing of the bath on the diffusion coefficients is not strong and slightly increases with T. The squeezing of the bath slightly change the values of D_{pp} and D_{qp} during the short transition time. With initial correlated bath the values of diffusion coefficients up to 50% larger than the ones for the uncorrelated bath at $\tilde{\omega}t<1.0$. Since the asymptotics of diffusion and friction coefficients are the same with correlated and uncorrelated bath subsystems, there is no practically difference in the collective energy

$$E_{coll}(t) = \xi(t)[\sigma_{qq}(t) + \langle q(t)\rangle^2]/2 + [\sigma_{pp}(t) + \langle p(t)\rangle^2]/(2\mu)$$

dissipation rate

$$\dot{E}_{coll}(t) = -2\lambda_p(t)E_{coll}(t) + [2\lambda_p(t)\xi(t) - \dot{\xi}(t)] \times(\sigma_{qq}(t) + \langle q(t)\rangle^2)/2 + D_{pp}(t)/\mu$$

in both cases. For the temperature $T=2\hbar\tilde{\omega}$, the relaxation times τ_R are about $2/\tilde{\omega}$ and $1.5/\tilde{\omega}$ at $\lambda_p=0.5\tilde{\omega}$ and $\lambda_p=2\tilde{\omega}$, respectively.

In Fig. 2 we present the time evolution of the variances of collective variables q and p taking the following initial conditions of the Gaussian distribution: $\langle q(0)\rangle=0.91(\mu\tilde{\omega}/\hbar)^{1/2}$, $\langle p(0)\rangle=0$, $\sqrt{\sigma_{qq}(0)}=0.06(\mu\tilde{\omega}/\hbar)^{1/2}$, $\sigma_{qp}(0)=0$, and $\sigma_{pp}(0)=\hbar^2/[4\sigma_{qq}(0)]$ (minimal uncertainty relation). The wave packet represents the coherent state of the oscillator. The curves for the correlated and uncorrelated heat baths reach the same asymptotics but differ from each other during some initial time interval. The variance σ_{qp} executes attenuated oscillations taking the positive and negative values and reaches finally zero. The values of σ_{pp} and σ_{qq} increases with increasing temperature T for given values of $\tilde{\omega}$, λ_p, and μ. We calculate the time dependence of the uncertainty relation (Fig. 3)

$$u(t) = \sigma_{pp}(t)\sigma_{qq}(t) - \sigma_{qp}(t)^2 - \hbar^2/4 = \sigma(t) - \hbar^2/4$$

and

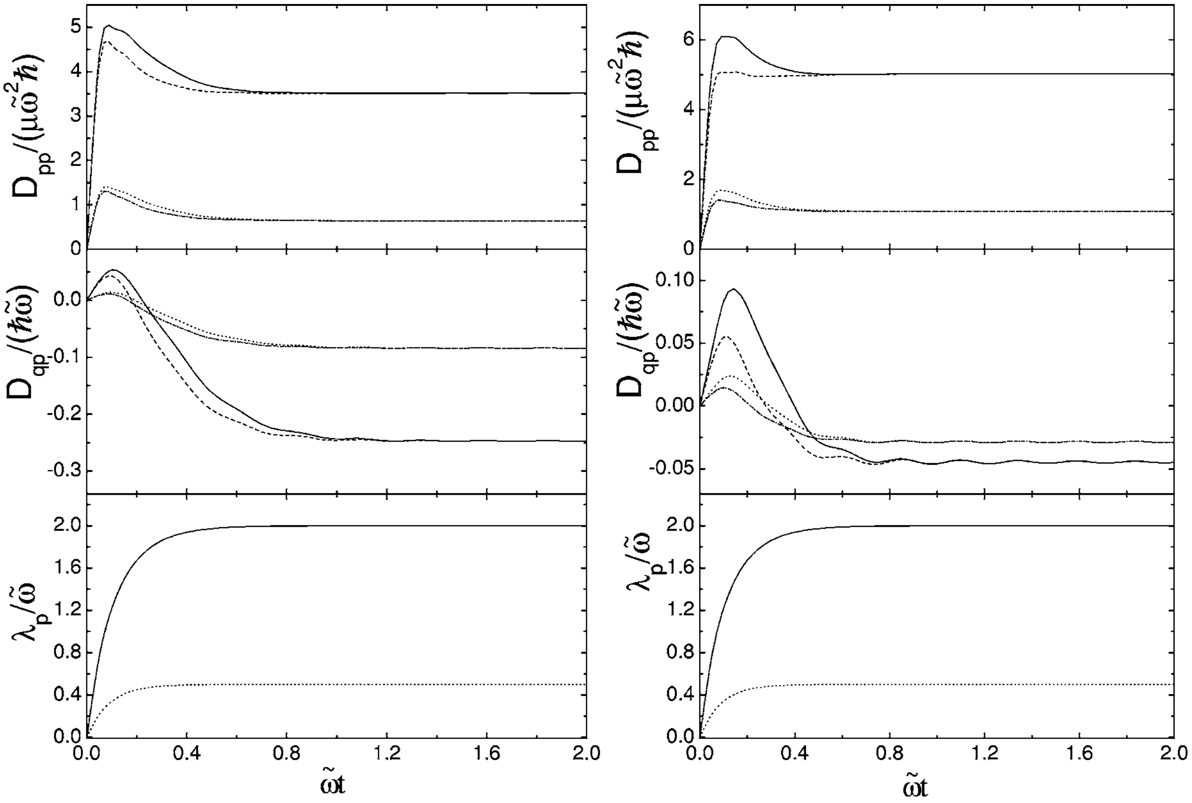

FIG. 1. Calculated time dependence of diffusion and friction coefficients for $\mu=448m_0$, $\hbar\tilde{\omega}=1$ MeV, $\phi_{\omega_0}=0$ at $T/\hbar\tilde{\omega}=1$ (left side) and $T/\hbar\tilde{\omega}=2$ (right side). The results for correlated (uncorrelated) thermal heat bath at $\lambda_p/\tilde{\omega}=0.5$ and $\lambda_p/\tilde{\omega}=2$ are presented by dotted (dash-dotted) and solid (dashed) lines, respectively.

$$\dot{u}(t)=\dot{\sigma}(t)=-2\lambda_p(t)\sigma(t)+2[D_{pp}(t)\sigma_{qq}(t)-2D_{qp}(t)\sigma_{qp}(t)].$$

The value of $u(t)$ is slightly larger for the correlated heat bath. The transient time is the same for the correlated and uncorrelated baths and increases with decreasing damping and increasing temperature. The absolute value of $u(t)$ grows about three to four times with increasing temperature from $T=\hbar\tilde{\omega}$ to $T=2\hbar\tilde{\omega}$. Since the rate of $u(t)$ or $\sigma(t)$ is explicitly connected with the rates of linear,

$$\dot{S}_{lin}=\hbar\dot{\sigma}(t)/[4[\sigma(t)]^{3/2}],$$

and von Neumann,

$$\dot{S}_N=\dot{\sigma}(t)\ln(1+2\hbar/\{2[\sigma(t)]^{1/2}-\hbar\})/\{2\hbar[\sigma(t)]^{1/2}\},$$

entropy production, the correlations in the heat bath play a minor role in S_{lin} and S_N. One can note that the time behavior of the nondiagonal components of the reduced density matrix is correlated with the time dependence of the entropy. Therefore there is correlation between $u(t)$ and the decoherence.

The following expression for the decoherence function [14]:

$$\tilde{G}(t)=-\frac{[\delta q]^2+4[\sigma_{qq}(0)]^2[\delta p]^2/\hbar^2}{8\sigma_{qq}(0)}\frac{J_{q_tq_t}}{\sigma_{qq}(t)},$$

$$\sigma_{qq}(t)=J_{q_tq_t}+A_t^2\sigma_{qq}(0)+\hbar^2B_t^2/[4\sigma_{qq}(0)]$$

can be used for all linear models with coordinate-coordinate coupling, Gaussian initial states, arbitrary coupling strengths, temperatures, and spectral densities. The decoherence function is expressed in terms of the initial width $\sigma_{qq}(0)$ of the superposed wave packets, and the initial separations δq and δp of the superposed wave packets in position and in momentum space, respectively. If we set $\sigma_{qq}(0)=\hbar/[2(\mu\tilde{\delta})^{1/2}]$, the superposed wave packets represent coherent states of the oscillator. The rate of change of $\tilde{G}(t)$ is $d\tilde{G}(t)/dt=-\frac{[\delta q]^2+4[\sigma_{qq}(0)]^2[\delta p]^2/\hbar^2}{4\mu\sigma_{qq}(0)}\frac{0.5(J_{q_tp_t}+J_{p_tq_t})\sigma_{qq}(t)-J_{q_tq_t}\sigma_{qp}(t)}{[\sigma_{qq}(t)]^2}$. Figure 4 shows that during transient time the decoherence function $G(t)=\frac{J_{q_tq_t}}{\sigma_{qq}(t)}$ (without the time-independent factor $-\frac{[\delta q]^2+4[\sigma_{qq}(0)]^2[\delta p]^2/\hbar^2}{8\sigma_{qq}(0)}$) depends weakly on the correlations in the heat bath. Obviously, $G(0)=0$ and $G(\infty)=1$ because $J_{q_0q_0}=0$ and $\sigma_{qq}(\infty)=J_{q_\infty q_\infty}$. The value of $\tilde{G}(t)$ or $G(t)$ is slightly larger for the correlated case. That is in correlation with the time behavior of $u(t)$. The remarkable result is that the decoherence time is practically the same for the different temperatures and does not decrease strongly with increasing damping. If $[\delta q]^2\approx 2\sigma_{qq}(\infty)$ and $[\delta p]^2\approx 2\sigma_{pp}(\infty)$, the time scales τ_D of the decoherence are about $1/\tilde{\omega}$ and $0.6/\tilde{\omega}$ at $\lambda_p=0.5\tilde{\omega}$ and $\lambda_p=2\tilde{\omega}$, respectively. One can see that for the nuclear system the effects of quantum interference are im-

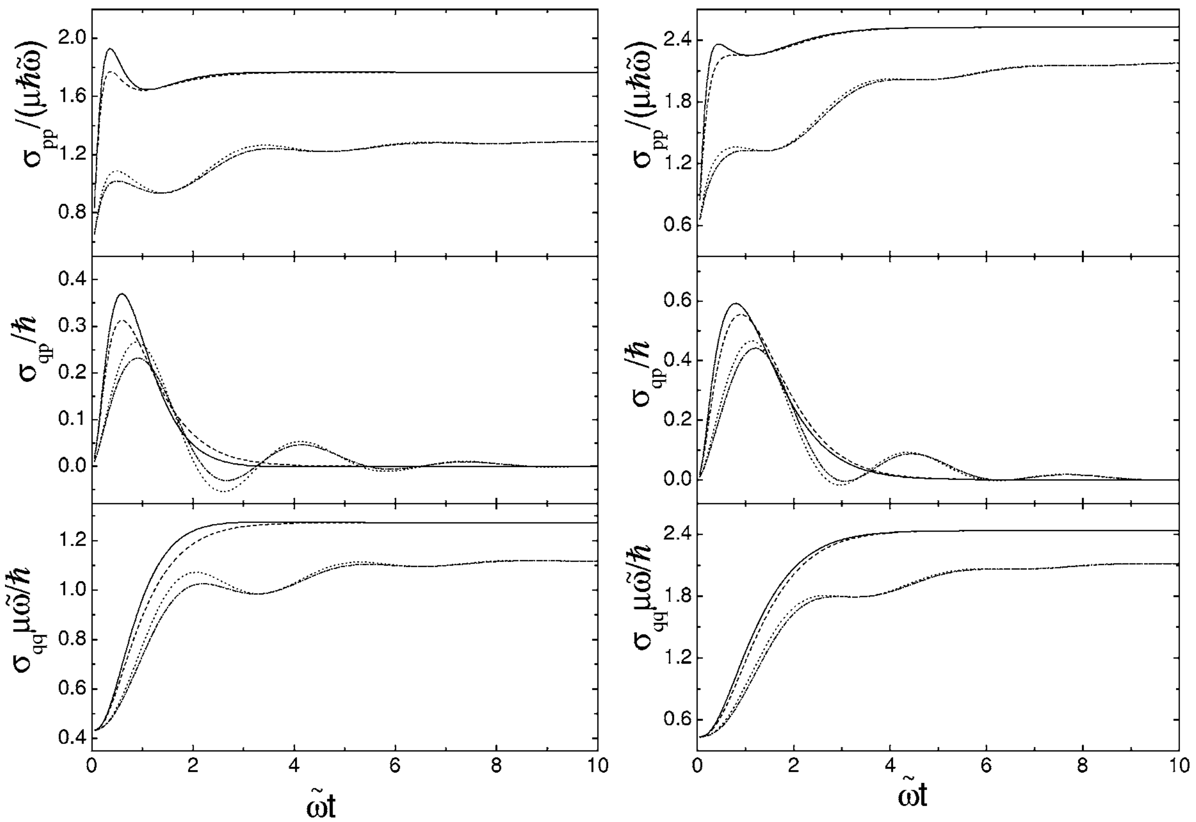

FIG. 2. The same as in Fig. 1, but for the variances.

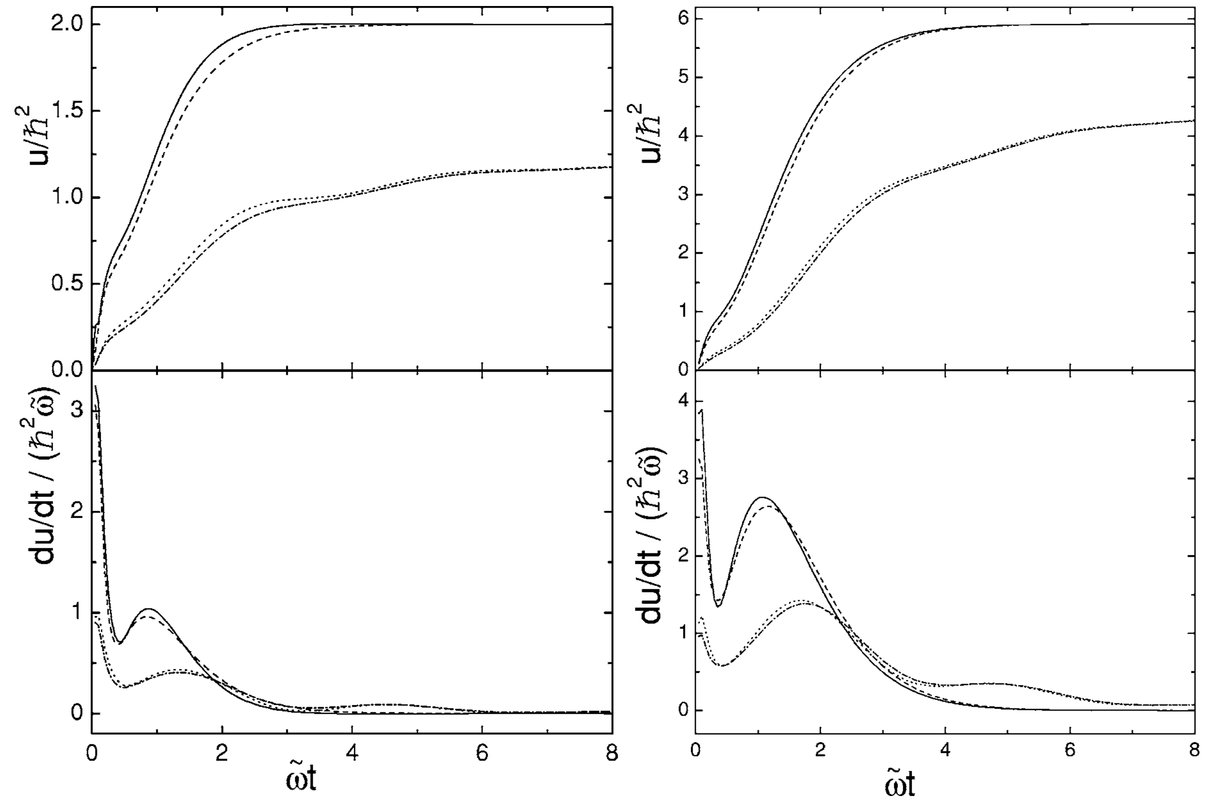

FIG. 3. The same as in Fig. 1, but for the uncertainty relation $u(t)$ and $\dot{u}(t)$. The initial variances are the same as in Fig. 2.

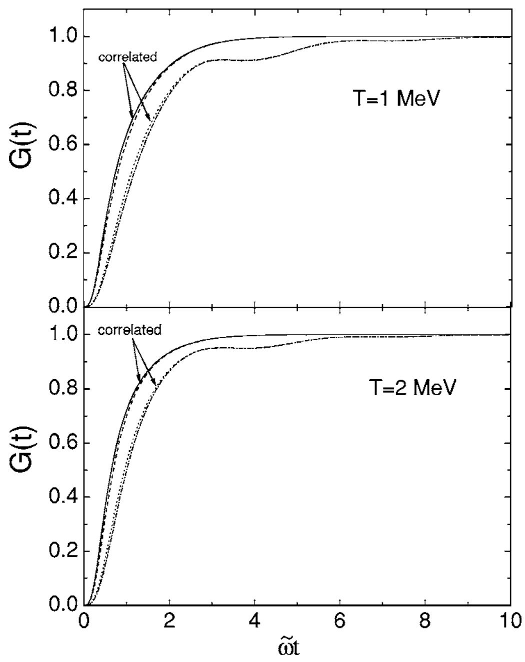

FIG. 4. Calculated time dependence of the decoherence $G(t)$ for $\mu=448m_0$, $\hbar\tilde{\omega}=1$ MeV, $\phi_{\omega_0}=0$ at $T/(\hbar\tilde{\omega})=1$ (upper part) and $T/(\hbar\tilde{\omega})=2$ (lower part). The results for correlated (uncorrelated) thermal heat bath at $\lambda_p/\tilde{\omega}=0.5$ and $\lambda_p/\tilde{\omega}=2$ are presented by dotted (dash-dotted) and solid (dashed) lines, respectively. The initial wave packet represents the coherent states of the oscillator (see text).

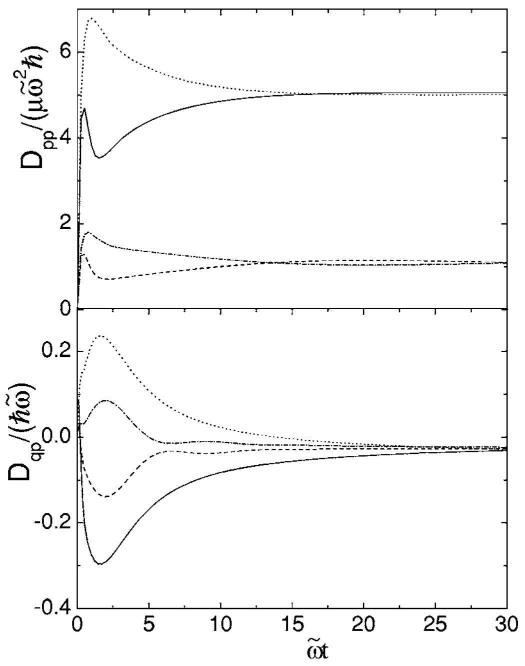

FIG. 5. Calculated time dependence of diffusion coefficients for $\mu=448m_0$, $\hbar\tilde{\omega}=1$ MeV, and $T/(\hbar\tilde{\omega})=2$. The results for correlated thermal heat bath are presented by dashed $(\phi_{\omega_0}=\pi/3)$ and dash-dotted $(\phi_{\omega_0}=-\pi/3)$ lines at $\lambda_p/\tilde{\omega}=0.5$, and by solid $(\phi_{\omega_0}=\pi/3)$ and dotted $(\phi_{\omega_0}=-\pi/3)$ lines at $\lambda_p/\tilde{\omega}=2$, respectively.

portant because τ_D and τ_R are comparable. As known already from the experiments, the coupling between different states is important in nuclear dynamics. For a typical macroscopic systems, $\tau_D \ll \tau_R$ [9,12].

B. The cases of $\phi_{\omega_0}=\pm\pi/3$

In the cases of $\phi_{\omega_0}=\pm\pi/3$ the transient times of diffusion coefficients (Fig. 5), specially for D_{qp}, variances (Fig. 6) and, correspondingly, uncertainty relation (Fig. 7) become larger in comparison to the cases of uncorrelated and correlated with $\phi_{\omega_0}=0$ heat baths (compare with Figs. 1 and 2). For $m_{\omega_0}\neq 0$, the transient times mainly increase due to the presence in Eq. (A1) the nonexponential in time terms which contain only B_0^i and B_0^i. One can see that the transition times weakly depend on λ_p. The calculated time evolution of the collective energy shows negligible difference between the cases of $\phi_{\omega_0}=0$ and $\phi_{\omega_0}\neq 0$. Thus the rate of dissipation of collective energy is almost independent of ϕ_{ω_0}. Figures 5–8 show that during transient time the absolute values of diffusion coefficients, variances, uncertainty relation $u(t)$ and the decoherence function become larger (smaller) in the case of $\phi_{\omega_0}=-\pi/3$ $(\phi_{\omega_0}=\pi/3)$, in comparison to the case of $\phi_{\omega_0}=0$. Since $D_{pp}(\phi_{\omega_0}=\pi/3)<D_{pp}(\phi_{\omega_0}=0)<D_{pp}(\phi_{\omega_0}=-\pi/3)$ during the initial transition time interval (see Figs. 1 and 4), the decay rate of quantum coherence increases at $\phi_{\omega_0}=-\pi/3$ and decreases at $\phi_{\omega_0}=\pi/3$ with respect to the uncorrelated and correlated (with $\phi_{\omega_0}=0$) heat baths. These results are in agreement with the results obtained by other methods in Refs. [4]. During the transient time the sign of D_{qp} depends on the sign of ϕ_{ω_0}. In the case of $\phi_{\omega_0}=\pm\pi/3$, the values of τ_D and τ_R are almost the same for the nuclear system considered.

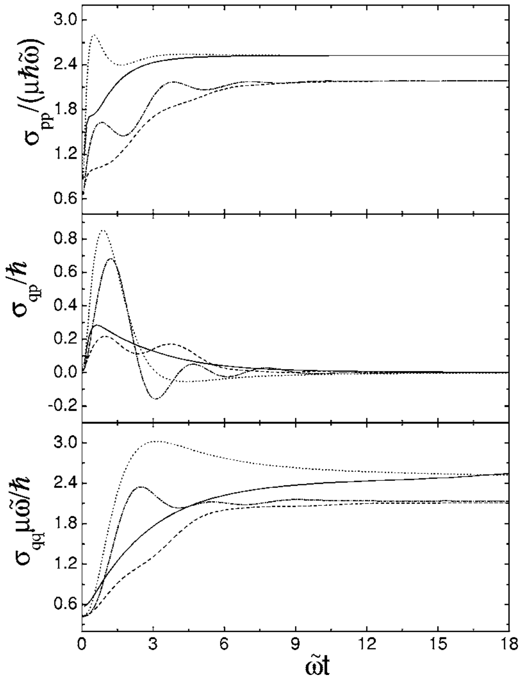

FIG. 6. The same as in Fig. 5, but for the variances. The initial variances are the same as in Fig. 2.

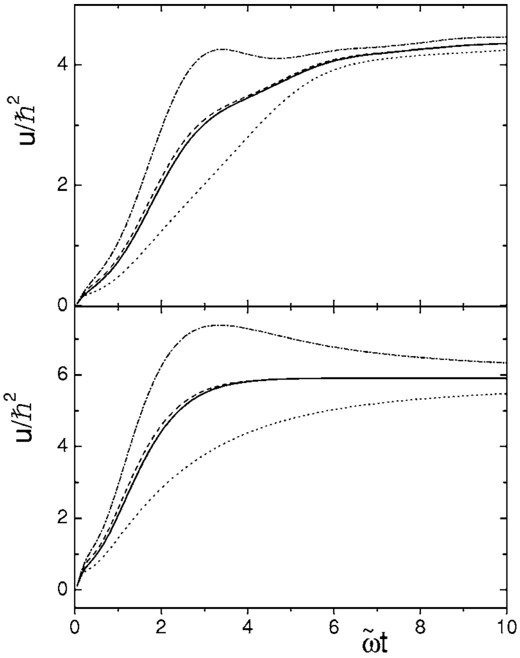

FIG. 7. Calculated time dependence of the uncertainty relation $u(t)$ for $\mu=448m_0$, $\hbar\tilde{\omega}=1$ MeV at $T/(\hbar\tilde{\omega})=1$ (upper part) and $T/(\hbar\tilde{\omega})=2$ (lower part). The results for correlated thermal heat bath are presented by dashed ($\phi_{\omega_0}=\pi/3$) and dash-dotted ($\phi_{\omega_0}=-\pi/3$) lines at $\lambda_p/\tilde{\omega}=0.5$, and by solid ($\phi_{\omega_0}=\pi/3$) and dotted ($\phi_{\omega_0}=-\pi/3$) lines at $\lambda_p/\tilde{\omega}=2$, respectively. The initial variances are the same as in Fig. 2.

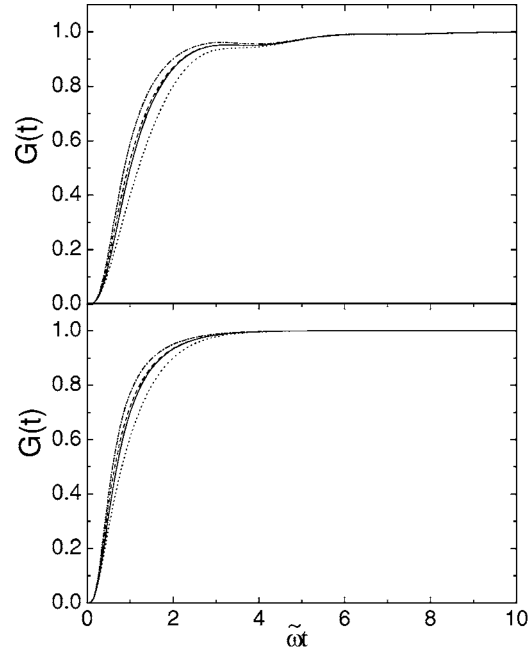

FIG. 8. The same as in Fig. 7, but for the decoherence function $G(t)$.

C. The case of $\phi_{\omega_0}=b\omega_0$

Let us consider the case of linear dependence of ϕ_{ω_0} on ω_0, i.e., $\phi_{\omega_0}=b\omega_0$ where $b\leq 0$. The main contributions to the squeezing parts $J^{(2)}_{q_tq_t}$, $J^{(2)}_{p_tp_t}$, and $J^{(2)}_{q_tp_t}$ of $J_{q_tq_t}$, $J_{p_tp_t}$, and $J_{q_tp_t}$ come from the terms which contain $B^i_0B^j_0\cos(b\omega_0+2\omega_0t)$, $B^i_0B^j_0\sin(b\omega_0+2\omega_0t)$, $B^i_0B^j_0\cos(b\omega_0+2\omega_0t)$, $B^i_0B^j_0\sin(b\omega_0+2\omega_0t)$, $B^i_0B^j_0\cos(b\omega_0+2\omega_0t)$, and $B^i_0B^j_0\sin(b\omega_0+2\omega_0t)$ because in comparison with other terms they do not decrease exponentially with increasing t (see the Appendix). Depending on the value of $b\omega_0+2\omega_0t$, these terms can compensate or heighten each other that creates the oscillations of $J^{(2)}_{q_tq_t}$ and

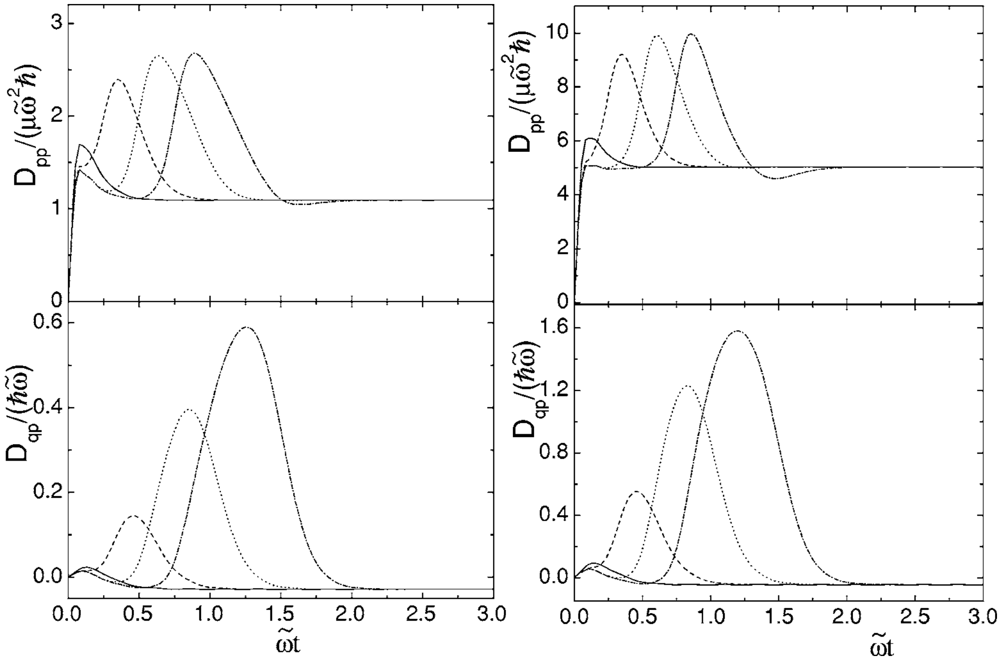

FIG. 9. Calculated time dependence of diffusion coefficients for $\mu=448m_0$, $\hbar\tilde{\omega}=1$ MeV at $\tilde{\omega}b=0$ (solid lines), -0.5 (dashed lines), -1.0 (dotted lines), and -1.5 (dash-dotted lines). The calculations are performed with $T/(\hbar\tilde{\omega})=2$, $\lambda_p/\tilde{\omega}=0.5$ (left side), and $\lambda_p/\tilde{\omega}=2$ (right side).

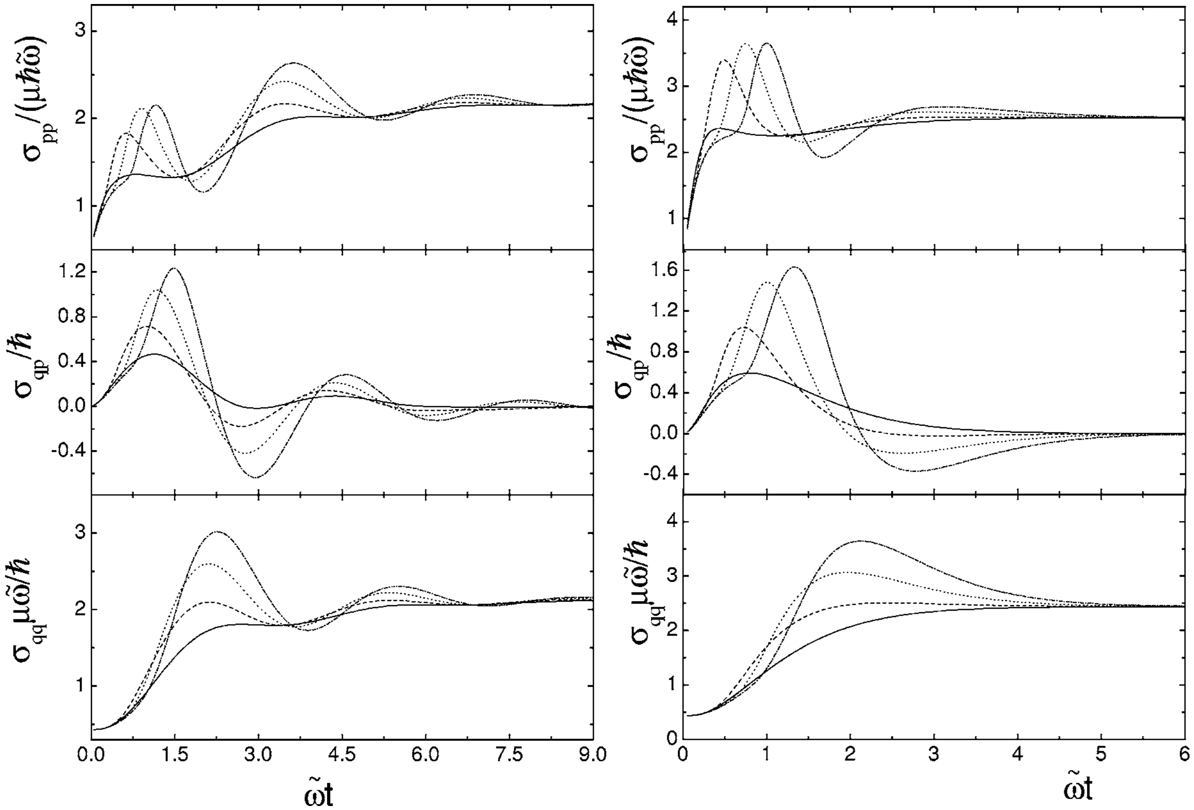

FIG. 10. The same as in Fig. 9, but for the variances. The initial variances are the same as in Fig. 2.

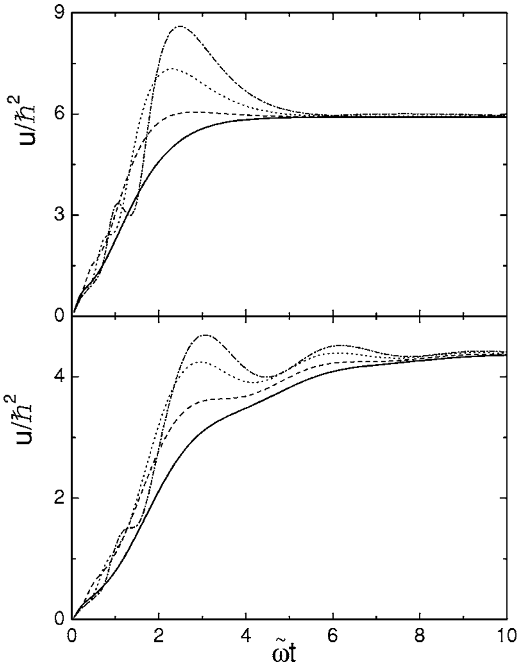

FIG. 11. Calculated time dependence of the uncertainty relation $u(t)$ for $\mu=448m_0$ and $\hbar\tilde\omega=1$ MeV at $\tilde\omega b=0$ (solid lines), -0.5 (dashed lines), -1.0 (dotted lines), and -1.5 (dash-dotted lines). The calculations are performed with $T/(\hbar\tilde\omega)=2$, $\lambda_p/\tilde\omega=0.5$ (lower part), and $\lambda_p/\tilde\omega=2$ (upper part). The initial variances are the same as in Fig. 2.

$J^{(2)}_{p_tp_t}$ during the transient time. Therefore the diffusion coefficients and variances in Figs. 9 and 10 oscillate as well. D_{ij} and σ_{ij} $(i,j=p,q)$ have maxima at $t\approx-b/2$ $(b\omega_0+2\omega_0 t\approx 0)$. At $\phi_{\omega_0}=b\omega_0$ the transitional effects in $J^{(2)}_{i_tj_t}$, D_{ij}, and σ_{ij} are shifted from the vicinity of $t=0$ to the vicinity of $t\approx-b/2$.

One can see in Fig. 11 that the oscillations in $u(t)$ increase with $|b|$. These oscillations occur above the curve corresponding to $b=0$. Therefore in comparison to the case of $b=0$ the quantum decoherence function $G(t)$ increases slightly faster with $b<0$ at $\tilde\omega t<3$ (Fig. 12).

In the case of $b>0$, there is no shift of the transitional effects from the vicinity of $t=0$. Therefore the difference of the calculated results at $b=0$ and $b>0$ is negligible.

IV. SUMMARY

The analytical expressions for the time-dependent transport coefficients were obtained for the FC oscillator and general coupling in coordinate between the collective subsystem and phase-sensitive squeezed heat bath. Since our Langevin approach is rigorous, it allows us to describe accurately the dependencies of transport coefficients, variances, uncertainty relation, and decoherence on the statistics of heat bath and to treat the open system beyond the weak-coupling and high temperature limits. The suggested approach is suitable for the problems of quantum optics, atomic, nuclear, and condensed matter physics.

The diffusion coefficients obtained with the correlated (at $\phi_{\omega_0}=0$) and uncorrelated heat baths differ slightly during the

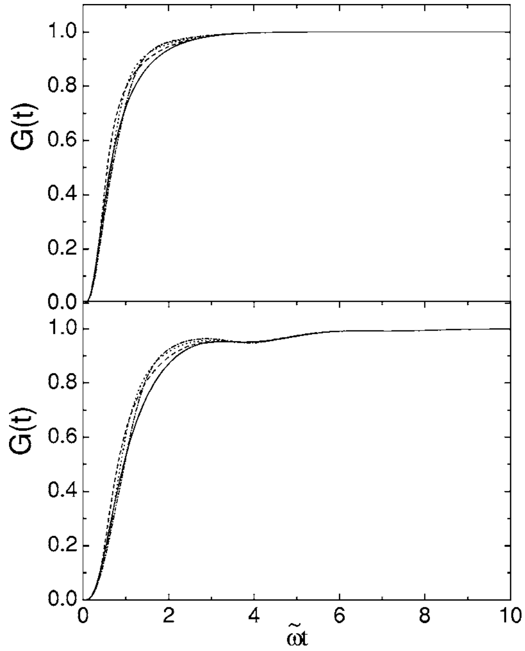

FIG. 12. The same as in Fig. 11, but for the decoherence function $G(t)$.

transient time but have the same asymptotic values. Hence the initially squeezed thermal bath leads to the minor changes of variances and dissipative properties of the collective system. As follows from the calculations, the dissipation of the collective energy is independent of the phases ϕ_{ω_0} in the correlated bath and differ slightly from the dissipation in the uncorrelated bath. At $\phi_{\omega_0}=0$ the decay rate of quantum coherence is slightly larger than the decay rate of the quantum coherence in the uncorrelated thermal bath. The decay rate of quantum coherence can be slightly enhanced or suppressed depending on the sign of phase of the ideally squeezed heat bath. In the cases of $\phi_{\omega_0}<0$ ($\phi_{\omega_0}>0$) the absolute values of diffusion coefficients, variances, and uncertainty relation become larger (smaller) than in the cases of uncorrelated and correlated with $\phi_{\omega_0}=0$ heat baths (Figs. 1 and 2). At $\phi_{\omega_0}\neq 0$ the transient times become larger than in case of $\phi_{\omega_0}=0$. As shown, the initial oscillations of the variances and diffusion coefficients are displaced from the vicinity of $t=0$ at $\phi_{\omega_0}=0$ to the vicinity of $t\approx|b|/2$ at $\phi_{\omega_0}=-|b|\omega_0$. This effect can be used for some applications. In this case the decay rate of quantum coherence slightly changes during the transitional time. For the uncorrelated and correlated heat baths, the values of decoherence and relaxation times are comparable for the nuclear systems.

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APPENDIX

The expressions for $J_{q_tq_t}$, $J_{p_tp_t}$, and $J_{q_tp_t}$ in Eqs. (13) and (14) for the diffusion coefficients are written for the correlated heat bath as follows:

$$J_{q_tq_t}=J^{(1)}_{q_tq_t}+J^{(2)}_{q_tq_t},$$

$$\begin{aligned}J^{(1)}_{q_tq_t}={}&\frac{2\hbar\omega\mu\lambda\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega_0\frac{\omega_0(2n_{\omega_0}+1)}{\gamma^2+\omega_0^2}\\&\times\{a^+_{ij}[(B^i_tB^j_t+B^i_0B^j_0)-(B^i_tB^j_0+B^i_0B^j_t)\cos(\omega_0t)]\\&-b^-_{ij}(B^i_tB^j_0-B^i_0B^j_t)\sin(\omega_0t)\},\end{aligned}$$

$$\begin{aligned}J^{(2)}_{q_tq_t}={}&\frac{4\hbar\omega\mu\lambda\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega_0\frac{\omega_0m_{\omega_0}}{\gamma^2+\omega_0^2}\{a^-_{ij}[B^i_tB^j_t\cos(\phi_{\omega_0})\\&+B^i_0B^j_0\cos(\phi_{\omega_0}+2\omega_0t)-(B^i_tB^j_0+B^i_0B^j_t)\cos(\phi_{\omega_0}\\&+\omega_0t)]+b^+_{ij}[(B^i_tB^j_0+B^i_0B^j_t)\sin(\phi_{\omega_0}+\omega_0t)\\&-B^i_0B^j_0\sin(\phi_{\omega_0}+2\omega_0t)-B^i_tB^j_t\sin(\phi_{\omega_0})]\},\end{aligned}$$

$$J_{p_tp_t}=J^{(1)}_{p_tp_t}+J^{(2)}_{p_tp_t},$$

$$\begin{aligned}J^{(1)}_{p_tp_t}={}&\frac{2\hbar\omega\mu^3\lambda\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega_0\frac{\omega_0[2n_{\omega_0}+1]}{\gamma^2+\omega_0^2}\{a^+_{ij}[(\dot{B}^i_t\dot{B}^j_t+\dot{B}^i_0\dot{B}^j_0)\\&-(\dot{B}^i_t\dot{B}^j_0+\dot{B}^i_0\dot{B}^j_t)\cos(\omega_0t)]-b^-_{ij}(\dot{B}^i_t\dot{B}^j_0-\dot{B}^i_0\dot{B}^j_t)\sin(\omega_0t)\},\end{aligned}$$

$$\begin{aligned}J^{(2)}_{p_tp_t}={}&\frac{4\hbar\omega\mu^3\lambda\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega_0\frac{\omega_0m_{\omega_0}}{\gamma^2+\omega_0^2}\{a^-_{ij}[\dot{B}^i_t\dot{B}^j_t\cos(\phi_{\omega_0})\\&+\dot{B}^i_0\dot{B}^j_0\cos(\phi_{\omega_0}+2\omega_0t)-(\dot{B}^i_t\dot{B}^j_0+\dot{B}^i_0\dot{B}^j_t)\cos(\phi_{\omega_0}\\&+\omega_0t)]+b^+_{ij}[(\dot{B}^i_t\dot{B}^j_0+\dot{B}^i_0\dot{B}^j_t)\sin(\phi_{\omega_0}+\omega_0t)\\&-\dot{B}^i_0\dot{B}^j_0\sin(\phi_{\omega_0}+2\omega_0t)-\dot{B}^i_t\dot{B}^j_t\sin(\phi_{\omega_0})]\},\end{aligned}$$

$$J_{q_tp_t}+J_{p_tq_t}=J^{(1)}_{q_tp_t}+J^{(1)}_{p_tq_t}+J^{(2)}_{q_tp_t}+J^{(2)}_{p_tq_t},$$

$$\begin{aligned}J^{(1)}_{q_tp_t}+J^{(1)}_{p_tq_t}={}&\frac{4\hbar\omega\mu^2\lambda\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega_0\frac{\omega_0(2n_{\omega_0}+1)}{\gamma^2+\omega_0^2}\{a^+_{ij}[\dot{B}^i_tB^j_t\\&-(\dot{B}^i_tB^j_0+\dot{B}^i_0B^j_t)\cos(\omega_0t)]-b^-_{ij}(\dot{B}^i_tB^j_0\\&-\dot{B}^i_0B^j_t)\sin(\omega_0t)\},\end{aligned}$$

$$\begin{aligned}J^{(2)}_{q_tp_t}+J^{(2)}_{p_tq_t}={}&\frac{8\hbar\omega\mu^2\lambda\gamma^2}{\pi}\sum_{ij}\int_0^\infty d\omega_0\frac{\omega_0m_{\omega_0}}{\gamma^2+\omega_0^2}\\&\times\{a^-_{ij}[\dot{B}^i_tB^j_t\cos(\phi_{\omega_0})+\dot{B}^i_0B^j_0\cos(\phi_{\omega_0}+2\omega_0t)\\&-(\dot{B}^i_tB^j_0+\dot{B}^i_0B^j_t)\cos(\phi_{\omega_0}+\omega_0t)]+b^+_{ij}[(\dot{B}^i_tB^j_0\\&+\dot{B}^i_0B^j_t)\sin(\phi_{\omega_0}+\omega_0t)-\dot{B}^i_0B^j_0\sin(\phi_{\omega_0}+2\omega_0t)\\&-\dot{B}^i_tB^j_t\sin(\phi_{\omega_0})]\},\end{aligned}\tag{A1}$$

where

$$a_{ij}^{\pm}=\frac{s_i s_j \pm \omega_0^2}{(s_i^2+\omega_0^2)(s_j^2+\omega_0^2)},$$

$$b_{ij}^{\pm}=\frac{(s_j \pm s_i)\omega_0}{(s_i^2+\omega_0^2)(s_j^2+\omega_0^2)}.$$

$J_{p_tp_t}$, $J_{q_tq_t}$, and $J_{q_tp_t}$ start from zero values at $t=0$ and in some time reach the asymptotic values which coincide with the asymptotic values of $J^{(1)}_{p_tp_t}$, $J^{(1)}_{q_tq_t}$, and $J^{(1)}_{q_tp_t}$. Note that the main contributions to $J^{(2)}_{p_tp_t}$, $J^{(2)}_{q_tq_t}$, and $J^{(2)}_{q_tp_t}$ give the terms which contain $B_0^iB_0^i$ and $B_0^iB_0^i$. For the uncorrelated heat bath, $m_{\omega_0}=0$ and $J_{p_tp_t}=J^{(1)}_{p_tp_t}$, $J_{q_tq_t}=J^{(1)}_{q_tq_t}$, and $J_{q_tp_t}=J^{(1)}_{q_tp_t}$ which correspond to the results of Refs. [19,26–30].

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